ular, we used the values 1500 m and 2000 m for the location of two reflectors; the values 1500, 2000, and 2600 m/sec for the three velocity values and a frequency range of 10-40 Hz. For these values, the depth to the first reflector is 10 wavelengths at the minimum frequency, while the separation between the layers is only 2.5 wavelengths.

The exact data generated by this scheme agree with the data generated by the Kirchhoff approximation to four decimal places. Thus, implementation on these exact synthetics would also be exact to four figures, since the method is exact for Kirchhoff data. Thus, the solid lines in Figures 1, 2, and 3 represent the output of the second method as well as the input.

Both methods were applied to field data provided to us by Lamont Observatory and used for demonstration purposes in Bleistein and Cohen (1982). Estimates of reflection strength on a major reflector at depth in that data set were consistent with estimates generated by other means. No onsite checks on the estimates are available.

References

Achenbach, J. D., 1973, Wave propagation in elastic solids: Elsevier, New York.

Bleistein, N., and Cohen, J. K., 1982, The velocity inversion problem: Present status, new directions: Geophysics, v. 47, p. 11.

Cerveny, V., Molotkov, I. A., Psencik, I., 1977, Ray methods in seismology: Univerzita Karlova, Praha.

Cohen, J. K., and Bleistein, N., 1979a, Velocity inversion procedure for acoustic waves: Gephysics, v. 44, p. 11.

——— 1979b, The singular function of a surface and physical optics inverse scattering: Wave motion, v. 1, p. 2.

Lahlou, M., Cohen, J. K., and Bleistein, N., 1983, Highly accurate inversion methods for three-dimensional stratified media: SIAM Applied Math, to appear.

Lewis, R. M., and Keller, J. B., 1964, Asymptotic methods for partial differential equations: The reduced wave equation and Maxwell's equations: New York University, Courant Institute of Mathematical Sciences, Research rep. EM-194.

Raz, S., 1981, A direct profile inversion: Beyond the Born model: Radio Sci., v. 16, p. 3.

Elastic Forward Modeling by the Fourier Method S7.2

Dan Kosloff, Moshe Reshef, and Dan Loewenthal, Tel-Aviv Univ., Israel

Elastic forward modeling is important both for the evaluation of conentional seismic reflection surveys, and for shear wave surveys. In the former, elastic effects are ignored and for different structures one constantly needs to evaluate the validity of this assumption. In shear surveys acoustic assumptions are no longer valid and full elastic modeling needs to be employed.

In elastic forward modeling, it is important to reproduce correct amplitudes of events since claims for the justification of ignoring elastic effects are often based on their small amplitudes. On the other hand the elastic wave equation is considerably more complicated than the acoustic wave equation, and therefore elastic forward modeling is more challenging. Thus with elastic modeling there is more justification for using direct methods such as finite differences which are expensive but are capable of producing correct amplitudes.

In this work we present an elastic forward modeling algorithim based on the Fourier method. The main advantage of the method is the high accuracy of the spatial derivative approximation. This allows for the separation of P waves from shear waves through application of divergence and curl numerical operators, and hence to follow the generation of converted waves. The numerical algorithim also allows for modeling with materials with high Poisson ratios and thus to approximate wave propagation in heterogeneous structures which contain both solids and fluids in juxtaposition.

In deriving the numerical algorithim, we depart from the approach of solving the vector wave equation for the displacements. Instead we derived a new set of equations for the stresses which does not include derivatives of the elastic constants. With this approach it becomes easier to affect the free surface boundary condition with the Fourier method and the algorithim appears more robust in modeling structures with severe velocity changes.

Basic equations

In a two-dimensional continuous medium, the linearized equations of momentum conservation are given by:

$$\rho f_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial_y} = \rho \ddot{U}_x,$$

and (1)

$$\rho f_y + \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial_y} = \rho \ddot{U}_y,$$

where x and y are Cartesian coordinates, σ_{xx}, σ_{yy}, and σ_{xy} are the three stress components, U_x and U_y represent the displacements, f_x and f_y represent the body forces, $\rho(x\ y)$ is the density. In (1) as in the remainder of this work, a dot above a variable represents a time derivative.

Under infinitesimal deformation, the twice differentiated in time strain displacement relations are given by:

$$\ddot{e}_{xx} = \frac{\partial \ddot{U}_x}{\partial x},$$

$$\ddot{e}_{yy} = \frac{\partial \ddot{U}_y}{\partial y}, \qquad (2)$$

and

$$\ddot{e}_{xy} = \frac{1}{2}\left(\frac{\partial \ddot{U}_x}{\partial y} + \frac{\partial \ddot{U}_y}{\partial x}\right),$$

where e_{xx}, e_{yy}, and e_{xy} represent the strain components.

After the substitution of (1) into (2), an alternative statement of momentum conservation is obtained;

$$\frac{\partial f_x}{\partial x} + \frac{\partial}{\partial x}\left[\frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}\right)\right] = \ddot{e}_{xx},$$

$$\frac{\partial f_y}{\partial y} + \frac{\partial}{\partial y}\left(\frac{1}{\rho}\left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}\right)\right) = \ddot{e}_{yy}, \qquad (3)$$

and

$$\frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} + \frac{\partial}{\partial y}\left(\frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}\right)\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho}\left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}\right)\right) = 2\ddot{e}_{xy}.$$

Downloaded 05 Jun 2012 to 72.20.129.98. Redistribution subject to SEG license or copyright; see Terms of Use at http://segdl.org/

Equation (3) contains the stresses and strains as unknowns whereas the displacements have been eliminated. In deriving this equation, no assumptions on material rheology were used and it also can be used for calculations for nonelastic media.

The three additional equations required for determining stresses and strains in the medium are supplied by the stress strain relation. In the simplest case of an elastic and isotropic medium, these relations are given by:

$$\begin{aligned}\sigma_{xx} &= (\lambda + 2\mu)e_{xx} + \lambda e_{yy}, \\ \sigma_{yy} &= \lambda e_{xx} + (\lambda + 2\mu)e_{yy}, \\ \sigma_{xy} &= 2\mu\, e_{xy}.\end{aligned} \tag{4}$$

λ and μ represent the Lame constant and the shear modulus respectively.

In a forward modeling problem, the geology of the modeled region is represented through the spatially variable material parameters λ, μ, and ρ. The seismic source is represented by the body forces f_x and f_y with appropriate time history. The solution gives the time histories of the stresses σ_{xx}, σ_{yy}, σ_{xy} at all points in space. From these stresses, values of other fields like the displacements can be generated.

Fourier method solution

In this study, equations (3) and (4) are solved by the Fourier method. The method includes a discretization of both space and time as with finite differences. The spatial derivatives in (3) are calculated with the aid of the fast Fourier transform (FFT) by using the property that a derivative in the spatial domain becomes a multiplication by $\sqrt{-1}$ times the wavenumber in the spatial frequency domain. The temporal derivatives in (3) on the other hand are calculated by second order differencing and the solution is stepped explicitly in time.

Introduction of the seismic source

The Fourier method allows flexibility in introducing seismic sources. Three types of sources can be used in the modeling, namely a directional force, a compressional force, and a shear source.

For the directional force, f_x (or f_y) is applied in a localized region with a bandlimited time history. The frequency band is chosen in order that all waves generated in the numerical mesh will be greater than or equal to the spatial Nyquist wavelength (two grid points). In the calculations, $\partial f_x/\partial x$, $\partial f_y/\partial y$, and $\partial f_x/\partial y + \partial f_y/\partial x$ are calculated by the discrete Fourier derivative approximation described previously. With a number of sources applied vertically on the surface, the vertical force can approximate an array of vibrators.

The compressional force is generated from a scalar potential $\phi(x,y)$ according to $f_x = \partial\phi/\partial x$ and $f_y = \partial\phi/\partial y$. The potential is applied locally in space with a band limited time history. The compressional source generates only P waves and thus imitates a symmetric explosion.

The shear source is generated from a function $\Psi(x,y)$ according to $f_x = \partial\Psi/\partial y$ and $f_y = -\partial\Psi/\partial x$. This source generates only shear waves. Pure shear sources are difficult to realize in practice, but in numerical modeling are sometimes useful in isolating and following shear wave propagation.

Strain compatibility

In using the three stresses σ_{xx}, σ_{yy}, and σ_{xy} as unknowns instead of the displacements U_x and U_y, it must be ensured that strain compatibility relations are not violated. It can be shown that in the continuous case, all the basic equations satisfy compatibility. For the Fourier approximation a discrete compatibility relation can be derived if all derivatives in equations (3) and (4) are calculated by the Fourier approximation. Violation of compatibility results in the presence of nonpropagating stresses.

Numerical examples

Results of applying the numerical algorithm to two important test problems are presented. The first example is of wave propagation in a homogeneous two-dimensional elastic half-space (Lamb's problem). The second example is of wave propagation in medium containing elastic and fluid half-spaces in contact. Both problems pose a challenge for numerical modeling techniques.

Two-Point Ray Tracing in 3-D Inhomogeneous Media S7.3

V. Pereyra, Univ. Central de Venezuela; and G. L. Wojcik, Weidlinger Associates

We describe an interactive numerical analysis code for 3-D ray tracing in completely inhomogeneous media with discontinuous velocities across curved interfaces. This is based on a two-point formulation (in contrast to the conventional initial value formulation of "shooting" codes) using a Newton iteration on a trial ray connecting source and receiver. In particular, the 3-D ray equations are expressed as a first-order system of nonlinear ordinary differential equations (ODEs) with algebraic boundary conditions, and solved using a variable order adaptive finite difference algorithm with global error estimates (PASVA4). Neighboring paths in a ray spread are found by continuation of the initial converged ray. For models with constant velocity between any two interfaces, the corresponding ray leg is linear, and the solver finds only the interface crossing points with no integration required. Velocity gradients, depending on their complexity, require one or more integration points and grid spacing is chosen automatically to equipartition the local error. These details are fortunately transparent to the user who interacts via a menu and graphics with a friendly front-end program tailored to a particular class of 3-D models—by specifying source and receiver locations and the ray code of interest, i.e., regions and interfaces traversed and wave type on each leg (P or S), as well as any modifications to the model.

There are a number of advantages to the two-point formulation in applications. First, the trial and error approach necessitated by shooting to connect source and receiver in complicated 3-D models is unnecessary; numerical iterations on the connecting ray are relegated to the two-point code's Newton solver—not the user (or additional software). Second, shooting is naturally included in the two-point approach by specifying source location, take-off angles and either traveltime, ray length, or the final interface, e.g., the free

Downloaded 05 Jun 2012 to 72.20.129.98. Redistribution subject to SEG license or copyright; see Terms of Use at http://segdl.org/