Gaussian Beam Migration

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Summary

We present a new migration method which utilizes the Gaussian beam approach. The method, though based on a high frequency approximation, operates on whole time sections and not merely on selected digitized horizons as in ray tracing map migration. However the method maintains many of the advantages of ray tracing such as the ability to migrate only certain propagation angles or to focus on selected regions without having to downward continue beneath the whole area of the survey.

In order to affect Gaussian beam migration, the recorded time section is first temporally transformed from the $x-t$ domain into the $x-\omega$ domain. The data is then beam stacked by the Gabor expansion (Raz, 1987). The stacking can be carried out over receiver coordinates, or over both receiver and shot coordinates. The expansion coefficients give amplitudes of Gaussian beams where each coefficient corresponds to a specific beam origin and beam angle. Each beam is then downward continued into the subsurface. The final migrated section consists of a sum of amplitude contributions from all beams at all frequencies.

Beam migration can be carried out on both CMP stacked sections as well as on prestacked data. For the latter, as in conventional prestack migration there is a possibility of migrating all shots separately or of downward continuing both shots and receivers.

Beam migration is demonstrated on a number of synthetic examples as well as on stacked field data. The results indicate that this migration is of high quality and offers a number of advantages over existing techniques.

The Gabor expansion and beam stacking

Given a function $f(x)$ which possesses a Fourier transform $\tilde{f}(k)$, its Gabor expansion is formally written as:

$$f(x) = \sum_{m,n} A_{mn}\, w_{mn}(x) \qquad m,n=0,\pm1,\pm2,\cdots \tag{1}$$

where,

$$w_{mn}(x) = w(x-mL)\, exp(in\Omega x) \ . \tag{2}$$

A_{mn} are coefficients to be determined, L and Ω are real positive constants related by,

$$\Omega L = 2\pi \ , \tag{3}$$

and $w(x)$ is a window function normalized according to:

$$\int_{-\infty}^{\infty} |w(x)|^2\, dx = 1 \ .$$

In the present study we use a Gaussian window function as originally proposed by Gabor (1946):

$$w(x) = \left(\frac{\sqrt{2}}{L}\right)^{1/2} \exp\left(-\pi\left(\frac{x}{L}\right)^2\right) \ . \tag{4}$$

The beam coefficients A_{mn} in (1) can be determined via the so called biorthogonal function γ_{mn} according to:

$$A_{mn} = \int_{-\infty}^{\infty} \gamma^*_{mn}(x)\, f(x)\, dx \ , \tag{5}$$

where

$$\int_{-\infty}^{\infty} w(x)\, \gamma^*_{mn}\, dx = \delta_{m0}\delta_{n0} \ ,$$

and,

$$\gamma_{mn}(x) = \gamma(x-mL)\exp(in\Omega x) \ . \tag{6}$$

Relations (1) and (5) represent the forward and inverse transformations analogous to the Fourier transform pair. There is also the possibility of representing $f(x)$ in (1) with the γ_{mn} functions and the A_{mn} functions in (5) with the $w(x)$ functions. This constitutes the dual Gabor representation (Raz, 1987).

The Gaussian beam migration uses the dual representation according to:

$$\tilde{P}(x,x_s,\omega) = \sum_{m,n} P_{mn}(x_s,\omega)\gamma^*_{mn}(x) \ ,$$

where $\tilde{P}(x,x_s,\omega)$ is the transformed time section corresponding to a shot location x_s with x the receiver coordinate.

The coefficients $P_{mn}(x_s,\omega)$ are given by,

$$P_{mn}(x_s,\omega) = \int_{-\infty}^{\infty} \tilde{P}(x,x_s,\omega)\, w(x-mL)\exp(in\Omega x)dx \ . \tag{7}$$

Relation (7) embodies the beam stacking procedure over receiver positions (Raz, 1987). In essence it consists of a series of spatial Fourier transforms of the product of the transformed time section and sliding window functions $w(x-mL)$.

The double beam stack is obtained after an additional transform over shot positions which yields:

$$P_J(\omega) = \int_{-\infty}^{\infty} P_{mn}(x_s,\omega)\, w(x_s - pL)\exp(iq\Omega x_s)\, dx_s, \quad (8)$$

Where J represents the four indices (m,n,p,q).

In implementing the beam stacking process condition (3) needs to be relaxed so that $\Omega L < 2\pi$. This creates an additional degree of freedom needed in the migration. The mathematical implications are given in Raz and Einziger (1988).

Common shot beam migration

The beam stacked data $P_{mn}(x_s,\omega)$ was shown to be interpretable as complex amplitudes of Gaussian beams emanating from nominal points $x = mL$ and with emergence angles $\phi_n = \sin^{-1}\left(\frac{n\Omega}{k}\right)$ where $k = \frac{\omega}{c}$ is the wavenumber with c the surface velocity. Since the propagation rules of Gaussian beams are well known (e.g Cerveny et al., 1982), this allows downward continuation of the beam stacked data.

Beam migration consists of downward continuation of all the generated beams and accumulation of their amplitudes in "pixels" in a spatial $x-z$ grid. The downward continued field is composed from contributions of beams emanating from all nominal points in all nominal directions at all frequencies. The final migrated section is obtained after application of an imaging condition to the downward continued field $\tilde{P}(x,z,\omega)$ as in conventional migration (Reshef and Kosloff ,1986).

Simultaneous migration of shot and receiver positions

This migration uses data which is doubly beam stacked with respect to receiver as well as shot positions. It therefore requires data with uniform shot spacing. It is assumed that both shot and receiver sampling are sufficiently dense to avoid spatial aliasing.

It was shown in Raz (1987) that the coefficients $P_J(\omega)$ in equation (8) correspond to a beam pair, with a receiver beam emanating from $x = mL$ at an angle $\phi_n = \sin^{-1}\left(\frac{n\Omega}{k}\right)$, and shot location at $x_s = pL$ at an angle $\phi_q = \sin^{-1}\left(\frac{q\Omega}{k}\right)$. Thus the migration is affected by downward continuation of both beam types and cumulating results in pixels over all beam pairs and over all frequencies. In reality a beam pair will significantly contribute only to pixels within a well defined neighborhood of the intersection of the receiver and shot beams.

Examples

The beam migration was tested against synthetic and field data. At first the features of the migration were evaluated with simple configurations such as a single diffractor in a uniform medium, a time section containing a single spike and planar dipping reflectors. Then more complicated synthetic examples were tested (e.g Fig 1-3). Finally CMP stacked field data were migrated.

These examples show that the beam migration produces high quality depth sections comparable to those of conventional high accuracy depth migration.

Fig 1 presents a synthetic salt dome model which was tested. The corresponding synthetic time section produced by a finite differences calculation is shown in Fig 2 (Freire, 1988). The migrated section is shown in Fig 3.

Conclusions

We have presented a new migration technique of beam stacked data. The examples tested show that this migration is capable of producing high quality results comparable to those of high accuracy conventional depth migration techniques. The results contain very few migration artifacts such as wraparound or edge effects.

Owing to the local nature of Gaussian beams there exists a possibility of downward continuing only selected beams and thus imaging a specific well defined region of interest. This advantage may become more important yet in three dimensional migration. From a computational viewpoint the migration can be performed efficiently however vectorization and parallelism may require more attention than with direct methods based on finite differences for which vectorization is natural.

The simultaneous migration of both receiver and shot coordinates is considerably faster than common shot migration and subsequent stacking of all shot images. It therefore may prove to be the more attractive alternative.

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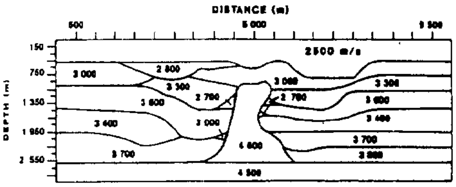

FIG. 1. Salt dome model configuration.

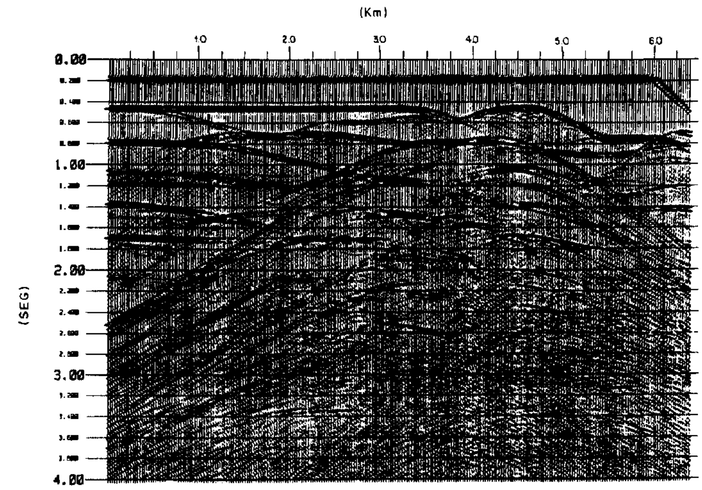

FIG. 2. Synthetic zero-offset time section for dome model.

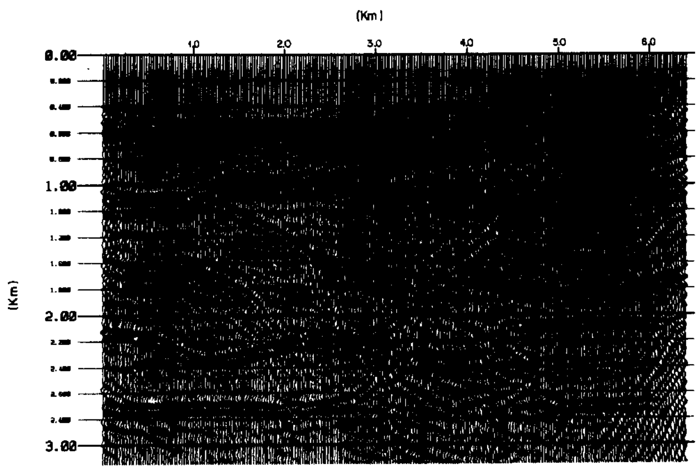

FIG. 3. Migrated depth section for dome model.