Long-Wave Anisotropy in Periodically Layered Media: A Numerical Test

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Summary

When a seismic signal propagates in a stratified earth there is anisotropy if the dominant wavelength is long enough compared to the layer thickness. In this situation, the layered medium can be replaced by an equivalent nondispersive transversely-isotropic medium. Theoretical and experimental analyses of the required minimum ratio of seismic wavelength to layer spacing based on kinematic considerations yield different results, with a much higher value in the experimental test.

The present work investigates the effects of layering by wave simulation and attempts to establish quantitatively the minimum ratio for which the long-wave approximation starts to be valid. We consider two-constituent periodically layered media, and analyze the equivalence for different material compositions and different material proportions in 1-D and 2-D media. Evaluation of the minimum ratio is done by comparing snapshots and synthetic seismograms visually and through a measure of coherence.

Layering induces scattering with wave dispersion or anisotropy depending on the wavelength to layer thickness ratio. The modeling confirms the dispersive characteristics of the wave field, the scattering effects in the form of coda waves, and the smoothed transversely-isotropic behaviour at long wavelengths. 1-D numerical tests for different media indicate that the minimum ratio is highest for the midrange of compositions and for stronger reflection coefficients between the constituents. For epoxy-glass the value is around $R = 8$, while for sandstone-limestone it is between $R = 5$ and $R = 6$. The 2-D case shows that, the more anisotropic the equivalent medium, the higher the minimum ratio, and that the approximation depends on the propagation angle with longer wavelengths required in the direction of the layering.

Introduction

In the earth, although a large part of the materials are intrinsically anisotropic, for instance many of the methamorphic and igneous rocks, their random orientation gives rise to a material which behaves isotropically when the dominant wavelength of the wave field is long compared to the crystal dimensions. However, due to the layered nature of sedimentary formations, transverse isotropy occurs when the dominant wavelength is long enough compared with the dimensions of the layers (e.g. Postma, 1955).

Wave propagation effects in stratified media depend on the wavelength of the signal. For wavelengths short compared to the dimensions of the layers, scattering in the form of coda waves is present, while the primary pulse shows a dispersive behaviour, i.e., the velocity is frequency dependent. On the other hand, at long wavelengths or low frequencies, the medium behaves as a non-dispersive smoothed transversely-isotropic material.

Helbig (1984) analyzed the dispersion relation for SH waves in a periodic layered medium and in the long-wave equivalent transversely-isotropic medium. He showed that the equivalence is valid for wavelengths larger than three times the spatial period of layering. An experimental test was carried out by Melia and Carlson (1984) where they made laboratory measurements of compressional wave velocities parallel and perpendicular to a periodic stratified medium consisting of glass and epoxy layers. They concluded that the long-wave approximation holds when the ratio R lies between 10 and 100, and that the minimum ratio is highest in the midrange of compositions, i.e., half glass and half epoxy.

Recently, numerical modeling has begun to be used as a tool for investigating the effects of layering on wave propagation (Carcione, (1987); Egan, (1989), Esmersoy et al., (1989); Kerner, (1989); and Philippe and Bouchon, (1989)). The present work is an attempt to establish quantitatively the minimum value of the ratio $R = \lambda_0/d$ for which a periodic layered medium can be replaced by a homogeneous transversely-isotropic medium, where λ_0 is the dominant wavelentht of the signal, and d is the spatial period of the stratified system. We restrict our analysis to a two constituent spatially periodic medium. In the following we shall refer to the periodic, isotropic, two-layered system as the PL medium, and the transversely-isotropic, long-wave equivalent medium as the TIE medium.

The analysis is performed by simulating Melia and Carlson's experiment, and wave propagation through the periodic limestone-sandstone sequence used by Postma to demonstrate the anisotropic properties of layering. These systems are representative of many solids in material science and of layered formations in the earth, and have dissimilar degrees of anisotropy and impedance constrasts. The simulations are carried out by solving numerically the wave equation for both the PL and the TIE media in 1-D and 2-D models.

We establish the minimum ratio R by comparison of the wave fields through a measure of coherence. The evaluation is made for different material proportions and for different values of the source dominant frequency (or different R values) for a given spatial period. Visual comparisons between snapshots and between synthetic seismograms are also used to establish the ratio qualitatively. The proposed numerical experiments take into account not only the kinematics but also the amplitude of the wave field. This evaluation can be considered conclusive, since the analysis with theoretical solutions is restricted by the complexity of the problem, and laboratory measurements are never exact due to experimental uncertainty.

1-D numerical tests

The wave velocities and densities of the isotropic materials are given in Table 1 with the respective P wave and S wave impedances. To calculate the average velocities we use the averaging technique developed by Backus (1962). The average velocities are tabulated in Table 2 for different material proportions, together with the average densities. Discretization of the space implies that $d_1 = n_1 DX$ and $d_2 = n_2 DX$, with DX the grid spacing, and n_1 and n_2 natural numbers. Thus, the spatial period is $d = (n_1 + n_2)DX$. By choosing, for instance, $d/DX = 4$ we obtain the different relative proportions indicated in Table 2. In the following examples the subindex nomenclature is $(1,2) \equiv (e,g)$ for the epoxy-glass periodic solid, and $(1,2) \equiv (l,s)$ for the limestone-sandstone periodic medium. The number of grid points is $NX = 385$, with $DX = 0.25\,mm$ grid spacing for epoxy-glass, and $DX = 25\,m$ for sandstone-limestone; therefore, the spatial periods are $d = 1\,mm$ and $d = 100\,m$ respectively. We define R as the ratio of dominant wavelenght λ_0 to spatial period d:

$$R = \frac{\lambda_0}{d}. \qquad (1)$$

where $\lambda_0 = \overline{V}/f_0$, with f_0 the central frequency of the source.

Snapshots and synthetic seismograms between the PL and the TIE media are compared by using the semblance. Figure 1 represents snapshots of the displacement field at $t = 15\,\mu s$ with the source located at grid point 192, for $R = 5$, and two different material proportions, (a) $P_g = 0.25$ and (b) $P_g = 0.50$. The dashed line is for the PL medium and the continuous line for the TIE medium. It can be seen that the best agreement is when $P_g = 0.25$. These results confirm that the long-wave approximation is a function of the proportions of the materials as found experimentally by Melia and Carlson (1984). They concluded that the minimum ratio was highest in the midrange of compositions ($P_g = 0.50$).

Figure 2 displays synthetic seismograms of the displacement field in a receiver located at grid point 142, for $P_g = 0.50$, where (a) $R = 3$, (b) $R = 5$ and (c) $R = 8$. The continuous line corresponds to the TIE media. The results for the semblance are summarized in Table 3, where the cases $P_g = 0.25$ and $P_g = 0.75$ are also included. As in the previous example, $P_g = 0.50$ gives the lowest values for the semblance. When $R = 3$, i.e. the dominant wavelength is three times the spatial period, the PL medium induces scattering which causes the coda waves, strongest for $P_g = 0.50$. Moreover, for differents values of R, the peak of the primary pulse has different arrival times. This means that the medium is dispersive, i.e. the velocity varies with wavelength. For this system, the long-wave approximation seems to be valid when $R > 8$ if we consider that a value greater than 97 % for the semblance makes the PL and TIE media equivalent. For illustration, Figure 3 represents the variation of the semblance with R for the case $P_g = 0.50$. As can be seen, the curve becomes flat at $R = 8$.

To study the influence of the density we consider a PL epoxy-glass system with $\rho_e = \rho_g = 1815\, Kg\, m^{-3}$ which is the average density when $P_g = 0.50$. The average velocity for this system when $P_g = 0.50$ is given in Table 2. The values of the semblance for $R = 3$, 5 and 8, are $S = 65$ %, 95 % and 99 % respectively. These results indicate that the minimium ratio is between $R = 5$ and $R = 6$, although it is not clear that this is due to the fact that the density is constant. The next example helps to clarify this point.

Let us consider a PL medium composed of sandstone and limestone whose properties are given in Table 1. The average velocity and density of the TIE medium for $P_l = 0.50$ are given in Table 2. The system is composed of alternating layers each $50\, m$ thick. Figure 4 displays synthetic seismograms for (a) $R = 3$, (b) $R = 5$ and (c) $R = 8$. As can be seen, the values of the semblance are similar to those obtained for constant density with the epoxy-glass system. Again, the long-wave approximation seems to be valid between $R = 5$ and $R = 6$. This difference to the variable density epoxy-glass system, for which the minimum ratio is around $R = 8$, suggests that the long-wave approximation depends on the impedance contrast between the constituents, i.e., on the reflection coefficients. From Table 1, $\Delta Z_P = 11122\ 10^3\, Kg\, s^{-1}\, m^{-2}$ and $\Delta Z_P = 5500\ 10^3\, Kg\, s^{-1}\, m^{-2}$ for variable and constant density epoxy-glass systems, respectively, and $\Delta Z_P = 7900\ 10^3\, Kg\, s^{-1}\, m^{-2}$ for the sandstone-limestone system. They give reflection coefficients of $R_P = 0.66$ (epoxy-glass), $R_P = 0.37$ (constant density epoxy-glass), and $R_P = 0.37$ (sandstone-limestone). These facts explain why the results are identical.

2-D numerical tests

The 2-D stratified medium is composed of alternating plane layers with thicknesses d_1 and d_2, whose material properties are given in Table 1. Using the averaging formulae from Backus (1962), we get the elasticities and density of the TIE medium for the different relative proportions. They are given in Table 4, where A is a coefficient of anisotropy for the quasi-P wave field, proportional to the difference in phase velocity along the symmetry axis and its normal direction (Melia and Carlson, 1984). It can be seen that the highest degree of anisotropy is around $P_g = 0.50$.

As in the 1-D case we define the ratios

$$R^{(P)} = \frac{\lambda_0^{(P)}}{d}, \qquad R^{(S)} = \frac{\lambda_0^{(S)}}{d}, \tag{2}$$

where $\lambda_0^{(P)} = \overline{V}_P/f_0$, and $\lambda_0^{(S)} = \overline{V}_S/f_0$ are the dominant wavelenghts, and $\overline{V}_P$ and $\overline{V}^S$ are the quasi-P and quasi-S energy velocities of the TIE medium.

We consider PL media with interfaces parallel to the X -axis. All the examples in this section use $NX = NZ = 405$, and a vertical source located at grid points (202,202). First we compute snapshots in an epoxy-glass system with $P_g = 0.50$ and spatial period $d = 1\, mm$. Grid spacing is $DX = DZ = 0.5\, mm$. The results for the u_x-component are displayed in Figures 5 and 6, for source dominant frequencies of $f_0 = 0.2\, MHz$ and $f_0 = 0.1\, MHz$, respectively, where (a) is the PL media and (b) is the TIE media. The propagating time is $27\, \mu s$. The values of the ratios depend on the angle θ between the symmetry axis and the propagation direction. For $f_0 = 0.2\, MHz$, they are $R^{(P)}(\theta = 0^\circ) = 13.44$, $R^{(P)}(\theta = 90^\circ) = 23.31$, and $R^{(S)}(\theta = 0^\circ) = R^{(S)}(\theta = 90^\circ) = 6.46$. For $f_0 = 0.1\, MHz$, these values should be multiplied by a factor of two. When $f_0 = 0.2\, MHz$, the P wave shows no differences between the PL and the TIE media, but the S wave has important differences around $\theta = 90^\circ$, while the cusps are perfectly equivalent. When $f_0 = 0.1\, MHz$, the two media are completely equivalent.

As mentioned before, the case $P_g = 0.50$ shows the highest degree of quasi-P wave anisotropy with $A = 26.8$ %. The 2-D sandstone-limestone TIE medium is less anisotropic with a value of $A = 9.8$ % for $P_l = 0.50$ as indicated in Table 4. To compare results between the two systems we compute snapshots in PL and TIE sandstone-limestone media. For $P_s = 0.50$ we take $DX = DZ = d_s = d_l = 10\, m$, i.e. a spatial period $d = 20\, m$, and a source dominant frequency $f_0 = 12\, Hz$. u_x-component snapshots at $0.48\, s$ are shown in Figure 7, where(a) is the PL media, and (b) the TIE media. The ratios are $R^{(P)}(\theta = 0^\circ) = 17.44$, $R^{(P)}(\theta = 90^\circ) = 14.31$, and $R^{(S)}(\theta = 0^\circ) = R^{(S)}(\theta = 90^\circ) = 7.88$, which are comparable with the values for the epoxy-glass test at $f_0 = 0.2\, MHz$ (see Figure 6). It is clear that in the latter case the long-wave approximation is not valid for the S wave, while it applies to the sandstone-limestone system as can be seen from Figure 7. Thus, the more anisotropic the TIE medium the higher the minimum ratio.

Conclusions

With the use of numerical modeling as a tool for simulating wave propagation in periodically layered media we conclude that:

- The modeling confirms that, depending on the relation of wavelength to layer thickness, a stratified medium induces dispersion, scattering and a smoothed transversely isotropic behaviour at long wavelengths.
- The minimum ratio of wavelength to layer thickness for the long-wave approximation to be valid is highest in the midrange of compositions, as found recently in laboratory experiments.
- The minimum ratio depends on material compositions through the reflection coefficients between the constituents. For instance, for epoxy-glass it is found to be around $R = 8$, and for sandstone-limestone, which has a lower reflection coefficient, it is between $R = 5$ and $R = 6$
- 2-D numerical tests reveal that the more anisotropic the equivalent medium, the higher the minimum ratio.
- The minimum ratio depends on the propagation angle. For S waves it is found that for a given value of wavelength to spatial thickness, the long-wave approximation applies for angles close to the symmetry axis and even to the cusps, but not in the direction of layering.

These tests were done in periodically layered media with isotropic constituents and, in the 2-D case, by using a vertical source. Further investigations should consider non-periodic media with anisotropic layers, and also different source types to study in detail the long-wave approximation for each wave mode.

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TABLE 1
Material properties of the PL media

Material	V_P (m/s)	V_S (m/s)	ρ $(Kg\,m^{-3})$	Z_P $(Kg\,s^{-1}m^{-2}\,10^3)$	Z_S $(Kg\,s^{-1}m^{-2}\,10^3)$
epoxy	2530	1200	1120	2833	1344
glass	5560	3200	2510	13955	8032
	2530	1200	1815	4591	2178
	5560	3200	1815	10091	5808
sandstone	2950	1620	2300	6785	3726
limestone	5440	3040	2700	14688	8208

TABLE 2
Material properties of the 1-D TIE media

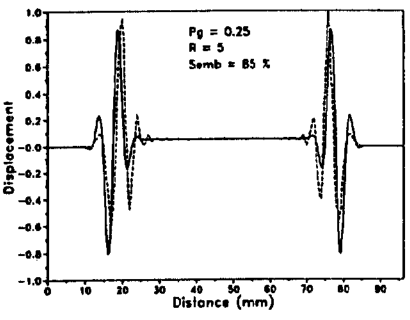
	P_2	n_1	n_2	$\bar{V}$ (m/s)	$\bar{\rho}$ $(Kg\,m^{-3})$
epoxy – glass	0.25	3	1	2513	1467
	0.50	2	2	2689	1815
$\rho(const)$	0.50	2	2	3256	1815
	0.75	1	3	3222	2162
sandstone – limestone	0.50	2	2	3578	2500

TABLE 3
Semblance (%) (epoxy-glass system)

		R = 3	5	8
P_g	0.25	56	91	98
	0.50	51	81	97
	0.75	56	83	98

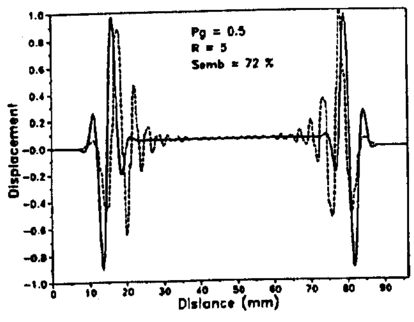
TABLE 4
Material properties of the 2-D TIE media

	P_2	n_1	n_2	$\bar{c}_{11}$ (GPa)	$\bar{c}_{13}$ (GPa)	$\bar{c}_{33}$ (GPa)	$\bar{c}_{55}$ (GPa)	$\bar{\rho}$ $(Kg\,m^{-3})$	A $(\%)$
epoxy – glass	0.25	3	1	23.2	4.6	9.7	2.1	1467	22.5
	0.50	2	2	39.4	5.8	13.1	3.0	1815	26.8
	0.75	1	3	56.2	8.7	22.4	5.4	2162	22.5
sandstone – limestone	0.50	1	1	47.5	12.3	32.0	9.7	2500	9.8



(a)

FIG. 1.



(b)

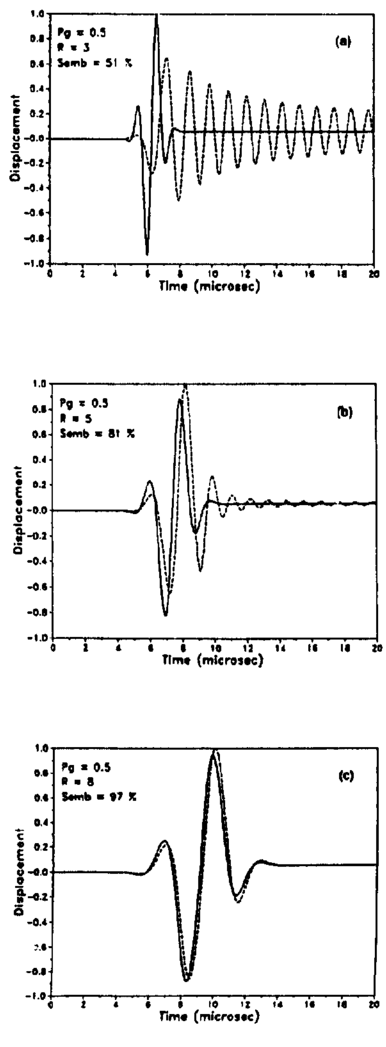


FIG. 2.

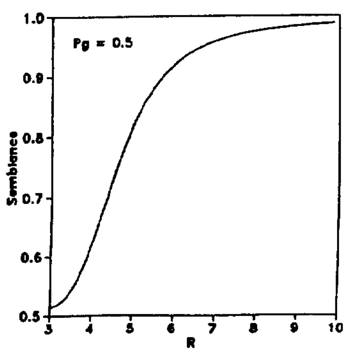


FIG. 3.

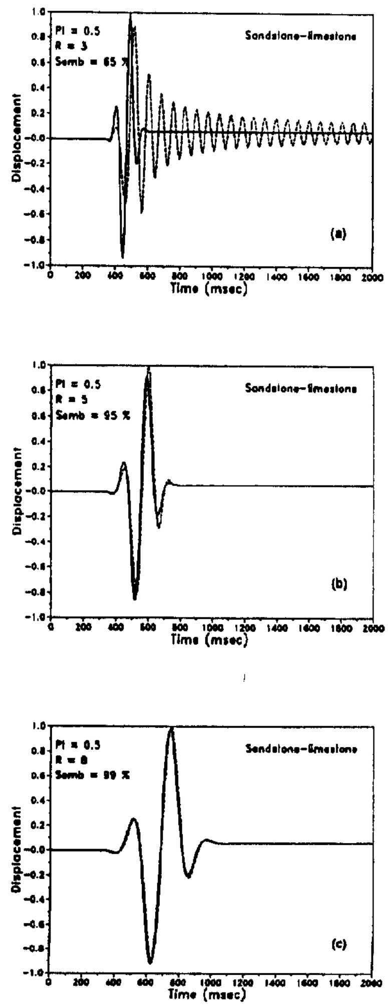
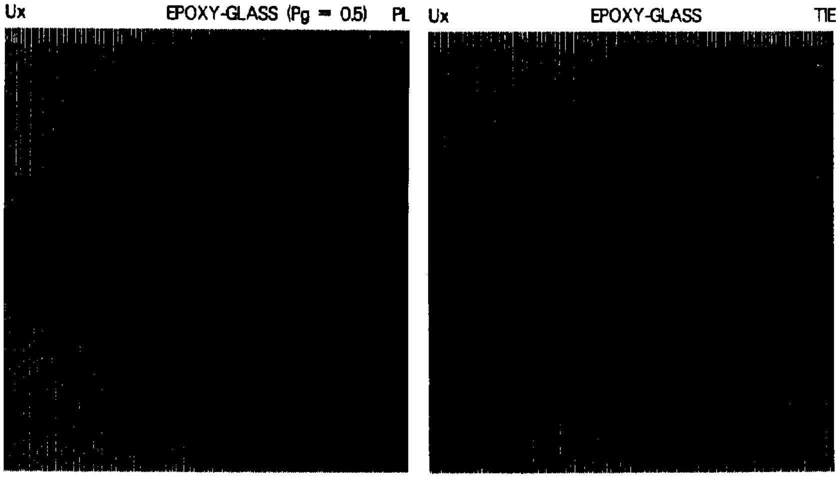
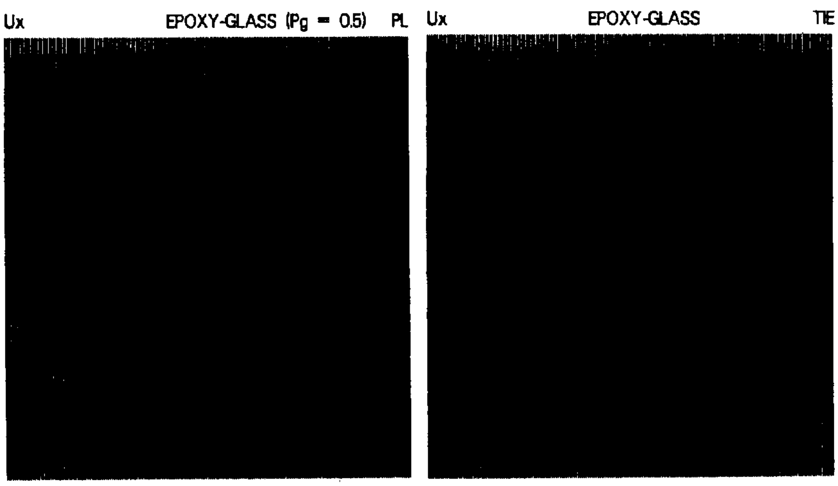


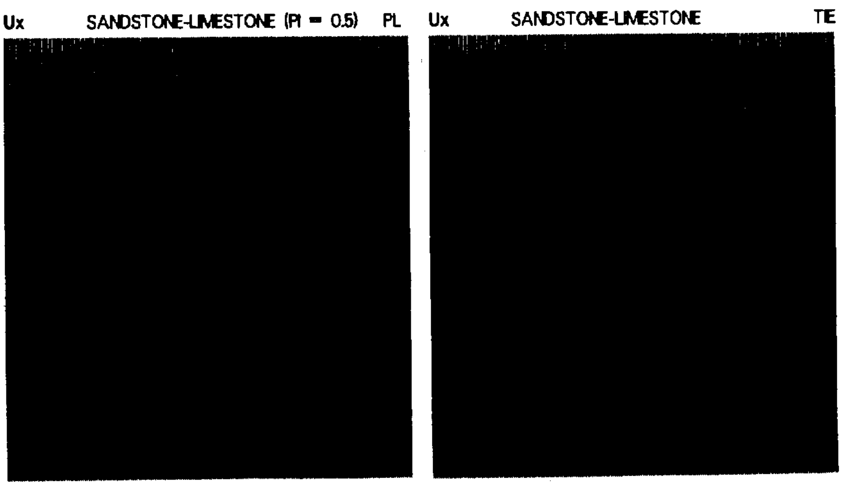
FIG. 4.



(a) FIG. 5. (b)



(a) FIG. 6. (b)



(a) FIG. 7. (b)

FIGURE CAPTIONS

FIG. 1. 1-D snapshots at $t = 15\,\mu s$ in epoxy-glass for $R = 5$, and two different material proportions, where (a) $P_g = 0.25$, and (b) $P_g = 0.50$. The dashed and continuous lines correspond to the PL and TIE media respectively.

FIG. 2. Synthetic seismograms in 1-D epoxy-glass for $P_g = 0.25$, where (a) $R = 3$, (b) $R = 5$, and $R = 8$. The dashed and continuous lines correspond to the PL and TIE media respectively.

FIG. 3. Semblance versus ratio of dominant wavelenght-spatial period for $P_s = 0.50$ in epoxy-glass.

FIG. 4. Synthetic seismograms in 1-D sandstone-limestone for $P_s = 0.25$, where (a) $R = 3$, (b) $R = 5$, and $R = 8$. The dashed and continuous lines correspond to the PL and TIE media respectively.

FIG. 5. u_x-component snapshots at $27\,\mu s$ propagation time in 2-D epoxy-glass for a source dominant frequency of $f_0 = 0.2\,MHz$, where (a) are the PL media, and (b) the TIE media. Thicknesses are $d_e = d_g = 0.5\,mm$. The dominant wavelenghts along the symmetry axis direction are $\lambda^{(P)} = 13.44\,mm$ and $\lambda^{(S)} = 6.46\,mm$.

FIG. 6. u_x-component snapshots at $27\,\mu s$ propagation time in 2-D epoxy-glass for a source dominant frequency of $f_0 = 0.1\,MHz$, where (a) are the PL media, and (b) the TIE media. Thicknesses are $d_e = d_g = 0.5\,mm$. The dominant wavelenghts along the symmetry axis direction are $\lambda^{(P)} = 26.88\,mm$ and $\lambda^{(S)} = 12.92\,mm$.

FIG. 7. u_x-component snapshots at $0.48\,s$ propagation time in 2-D sandstone-limestone for a source dominant frequency of $f_0 = 12\,Hz$, where (a) are the PL media, and (b) the TIE media. Thicknesses are $d_s = d_l = 10\,m$. The dominant wavelenghts along the symmetry axis direction are $\lambda^{(P)} = 349\,m$ and $\lambda^{(S)} = 157\,m$.