PRACTICAL SOLUTION OF THE 3D EIKONAL EQUATION ON A MIXED GRID

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Introduction

Calculation of 3D traveltimes is needed for the Kirchoff integral based prestack depth migration. 3D prestack imaging requires a significant large amount of computation where a major part is dedicated to traveltime calculations. Therefore, a practical way for calculating traveltimes is needed.

The algorithm presented here was developed according to the following objectives: (a) the numerical scheme should be simple and efficient, (b) traveltimes should represent arrival times of body waves, and (c) the calculated traveltime field should be accurate for steep dip structures.

To achieve the objectives, we: (a) directly solve the 3D Eikonal equation on a cartesian coordinate system, (b) modify the over-critical arrivals on a special cylindrical coordinate systems, and (c) apply a spatial convolutional operator to overcome errors introduced by numerical approximations.

Solution scheme

Propagation starts by downward extrapolating the time field T(x,y,z) from depth z to depth z+dz. The Eikonal equation is directly solved on a cartesian coordinate system G(x,y,z) (Reshef and Kosloff, 1986). The resultant traveltime field T(x,y,z+dz), contains arrival times of body-waves as well as of headwaves. In order to replace the travetimes of the (first to arrive) head-waves with the appropriate bodywaves (direct arrivals), we transform T(x,y,z+dz) to a special cylindrical coordinate system $H(r,\theta,z)$, where we can follow the direction of propagation. The cylindrical coordinate system $H(r, \theta, z)$ is constructed using a local radius r and a local angle θ . θ is the angle of propagation of the wave fronts on a grid node of G, and r is the distance from a grid node of G to the intersection with the neighboring grid cell. Next, we locate all the r, θ nodes of H where head-waves arrivals exist and remove the traveltime information belong to these nodes. Similar to using the upwind condition (Van Trier and Symes, 1988), we use the transformed traveltime field $T(r, \theta, z+dz)$, and stretch the solution along each local radius back to the above x, y node on G. The modified traveltime function $T'(r, \theta, z+dz)$ contains now arrival times associated with body-waves only. The traveltime field T' might contain some noise, resulting from numerical approximations. Therefore, as a final step, we apply a 2D spatial convolutional operation, aiming to correct local inconsistencies. A 3D model and resulting traveltime cube are shown in figures 1 and 2.

References

Reshef M. and Kosloff D., 1986, Migration of common shot gathers: Geophysics, 51, 324-331. Van Trier J. and Symes W., 1991, Upwind finite-differences calculation of traveltimes: Geophysics, 56, 812-821.

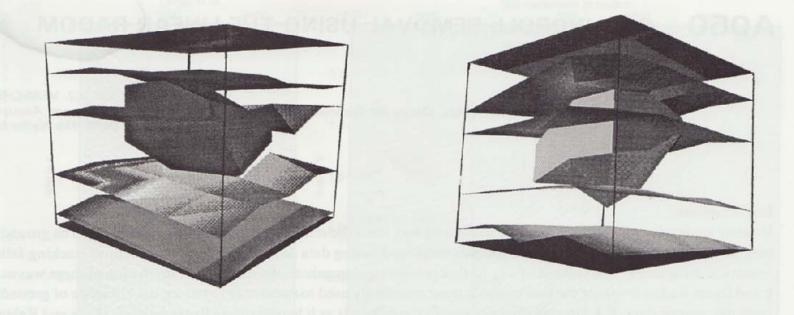


Figure 1: Two views of a 3D model. The model consists of a salt body and four semi-horizontal layers.

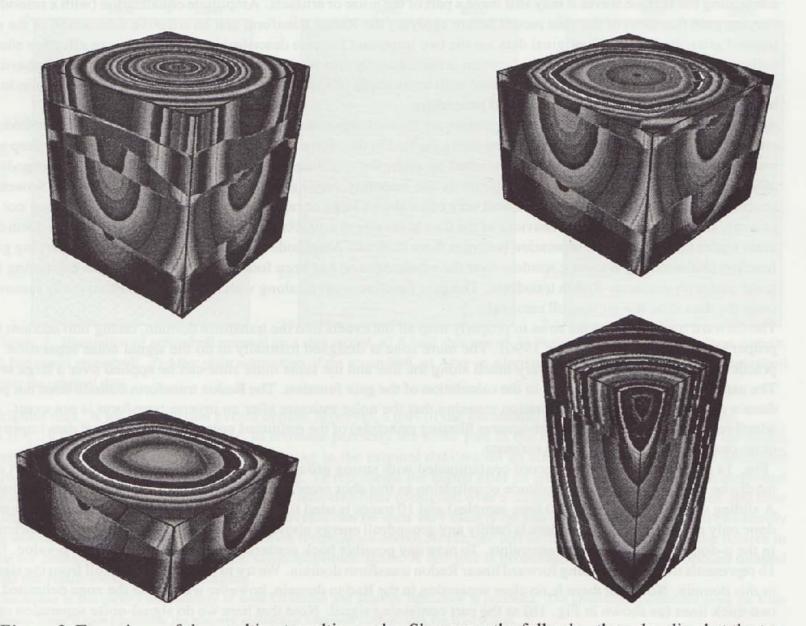


Figure 2: Four views of the resulting traveltime cube. Shown are the full cube, the cube sliced at the top of the salt level, the cube sliced at the bottom of the salt level, and the cube sliced at the center. Note that the traveltime field is not necessarily continuous.