Numerical Solution of the Constant Density Acoustic Wave Equation by Implicit Spatial Derivative Operators

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Summary

A new numerical scheme for the solution of the constant density acoustic wave equation is derived. The scheme is based on recursive second derivative operators which are derived by an L_∞ fit in the spectral domain. The use of recursive derivative operators enables to extend the forward modeling to shorter wavelengths. An example of reverse time migration of a synthetic dataset shows that the numerical dispersion can be significantly reduced with respect to schemes based on finite differencing.

Introduction

The numerical solution of the acoustic wave equation is routinely used for generating synthetic seismic surveys. These simulations are also the basis of reverse-time migration.

The accuracy and efficiency of the numerical simulation depends on the method used for approximating the spatial derivatives in the wave equation. The fastest and least accurate method is second order finite differences while the pseudo spectral method is more accurate but considerably slower. In general there is a trade-off, where for a specified accuracy, a lower order method will require a larger number of grid points with a small number of calculations per point, whereas a higher order method will require less grid points but with more calculations per point. Experience indicates that the fourth order or sixth order finite difference schemes yield the best trade-off. For high order schemes there is a saturation effect where the improvement in accuracy diminishes and does not justify the added computational effort.

This study examines the use of recursive derivative operators for improving the efficiency of the numerical simulation. The application of these operators requires the solution of tri-diagonal linear equation systems which can be carried out efficiently. The derivative operators are designed by a Remez exchange procedure.

In the following we describe the implicit spatial derivative operators and compare their accuracy to the accuracy of explicit derivative operators. We then present a numerical example which compares simulations with implicit operators to fourth order finite difference simulations and pseudo spectral numerical simulations.

Implicit Derivative Operators

Given a function $f(x)$, we denote its sampled values by $f[j] = f(x = jdx)$. The recursive second derivative approximation writes,

$$\frac{d^2 f}{dx^2}[j] = \frac{a_0 + a_1\Delta_1 + a_2\Delta_2 + ... + a_N\Delta_N}{1 + b_1\Delta_1 + b_2\Delta_2 + ... + b_M\Delta_M} f[j], \qquad (1)$$

where $\Delta_k f[j] = f[j+k] + f[j-k]$.

We will consider operators for which $M \leq N$. (1) can be recast in an equivalent form,

$$\frac{d^2 f}{dx^2}[j] = (c_0 + ...c_{N-M}\Delta_{N-M} + \frac{d_0}{1+\beta_0\Delta_1} + ... + \frac{d_{M-1}}{1+\beta_{M-1}\Delta_1})f[j] \quad (2)$$

The coefficients in (2) can be related to the coefficients in (1). Equation (2) is more convenient for calculations while (1) is more suitable for the design of the coefficients. The first terms in (2) $c_0 + ...c_{N-M}\Delta_{N-M}$ forms an explicit operator. Each of the terms $\frac{d_j}{1+\beta_j\Delta_1} f[j]$ is a tri-diagonal equation system.

Operator Design

The coefficients a_l and b_l in (1) are calculated by an L_∞ norm fit in the spectral domain. The resulting equations are given by,

$$a_0 + 2a_1\cos k_L dx + 2a_2\cos 2k_L dx + ... + 2a_N \cos Nk_L dx +$$
$$+2b_1 k_L^2 \cos k_1 dx + ... + 2b_M k_L^2 \cos Mk_L dx +$$
$$+(-1)^L e = -k_L^2 \quad L = 1........N+M+2. \qquad (3)$$

The unknowns are $a_0, a_1........a_N, b_1.........b_M$ and the error e. The $N+M+2$ wave number components k_L are within the range $0 \leq k_L < k_{\max} < \pi dx$, where $k_{\max}$ is specified by the user. This value should be set to give the best compromise between the objectives of accuracy of the fit and the shortest wavelength which can be propagated with little numerical dispersion. The system (3) is solved iteratively, where each time the values of k_L are selected at the locations of the extrema of the error.

Accuracy of the derivative operators

We consider the application of the spatial derivative operator to the function $f[j] = e^{ik\,jdx}$ for different k values, and denote the output as $-\bar{k}^2 f[j]$. The normalized numerical phase velocity is given by $c_f = k/\bar{k}$. The normalized phase velocity is plotted against wave number in Fig 1 for a number of different operators. In Fig 1, fd-4 represents the fourth order finite differences operator, whereas 3-1, for example, denotes an operator obtained from (3) with N=3 and M=1 respectively. In the ideal case the operator would yield a phase velocity value of 1 up to the Nyquist wave number $k_f dx = \pi$. In the design of the operators, the maximum wave number $k_{\max}$ was adjusted so that the maximum normalized phase velocity error in the range $0 \le k_L \le k_{\max}$ becomes less that 0.5%.

Examination of Fig 1 shows that the inclusion of an implicit term in the derivative operator significantly improved the accuracy. In particular the 3-1 operator produces a very good response. A second observation is that the 3-0 operator has a better response than the fourth order finite difference operator which has the same number of coefficients.

Example

The numerical derivative operators were tested in a 2D example of zero offset reverse time migration. The subsurface model consisted of a single reflector with two segments dipping in opposite directions and one horizontal segment. The model had a constant velocity of 4500 m/sec. The peak frequency in this example was 25Hz, the horizontal and vertical grid spacing was 27m, and the time step size was 2msec. The input time section was obtained by numerical modeling using the program Susylnv from the SU Package.

Fig 2 a-d respectively shows migrated images obtained with the fourth order finite difference method, the 2-1 operator method, the 3-1 operator method, and the pseudo spectral method. As shown in Fig 2, the numerical dispersion is most prominent in Fig 2a of the fourth order finite difference method. Fig 2b of the 2-1 implicit method is somewhat improved while the improvement in Fig 2c of the 3-1 implicit method is very obvious. In fact the result in Fig 2c is almost as good as the result of the pseudo spectral method in Fig 2d.

Conclusions

We have presented a new solution scheme for the acoustic wave equation. The method is based on implicit derivative operators. It was shown that these operators enable to extend forward modeling to shorter wavelengths in an efficient manner.

The application of the implicit derivative operators involves solution of tri diagonal linear equation systems. Such systems can be solved with approximately 2N multiplications where N is the number of equations. This value is about twice the number of operations which are required for each explicit term.

The method of this study also applies to 3D. A similar approach can also be applied to the design of staggered grid first derivative operators.

Figure 1 Normalized phase velocity versus normalized wave number $k'dx = \frac{k\,dx}{\pi}$ for different second derivative operators.

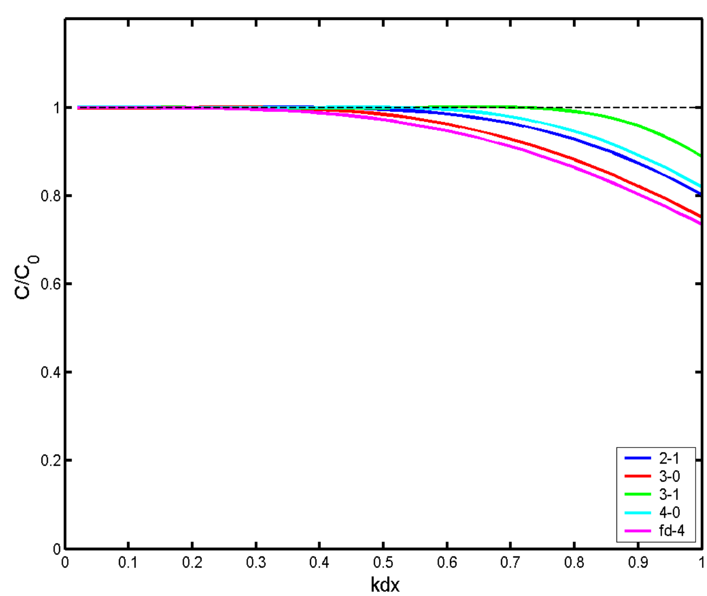


Figure 2 – Zero offset time section

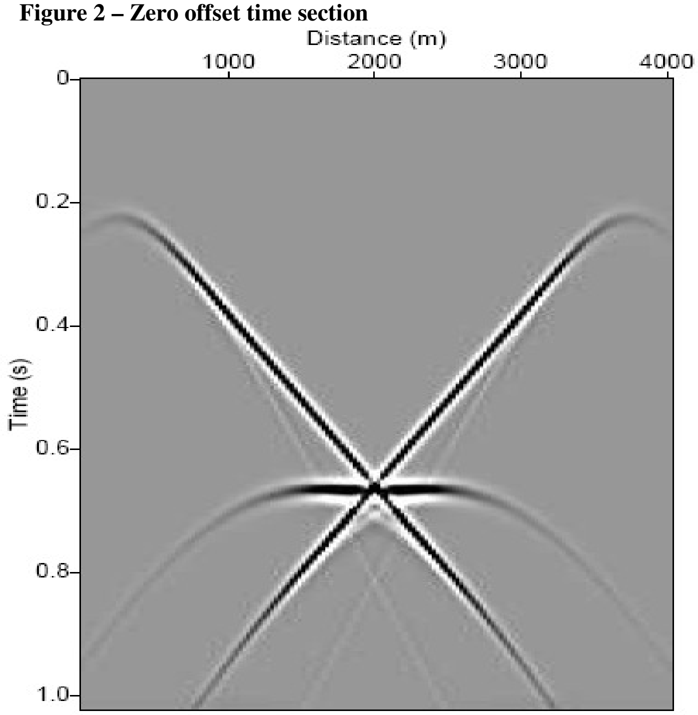
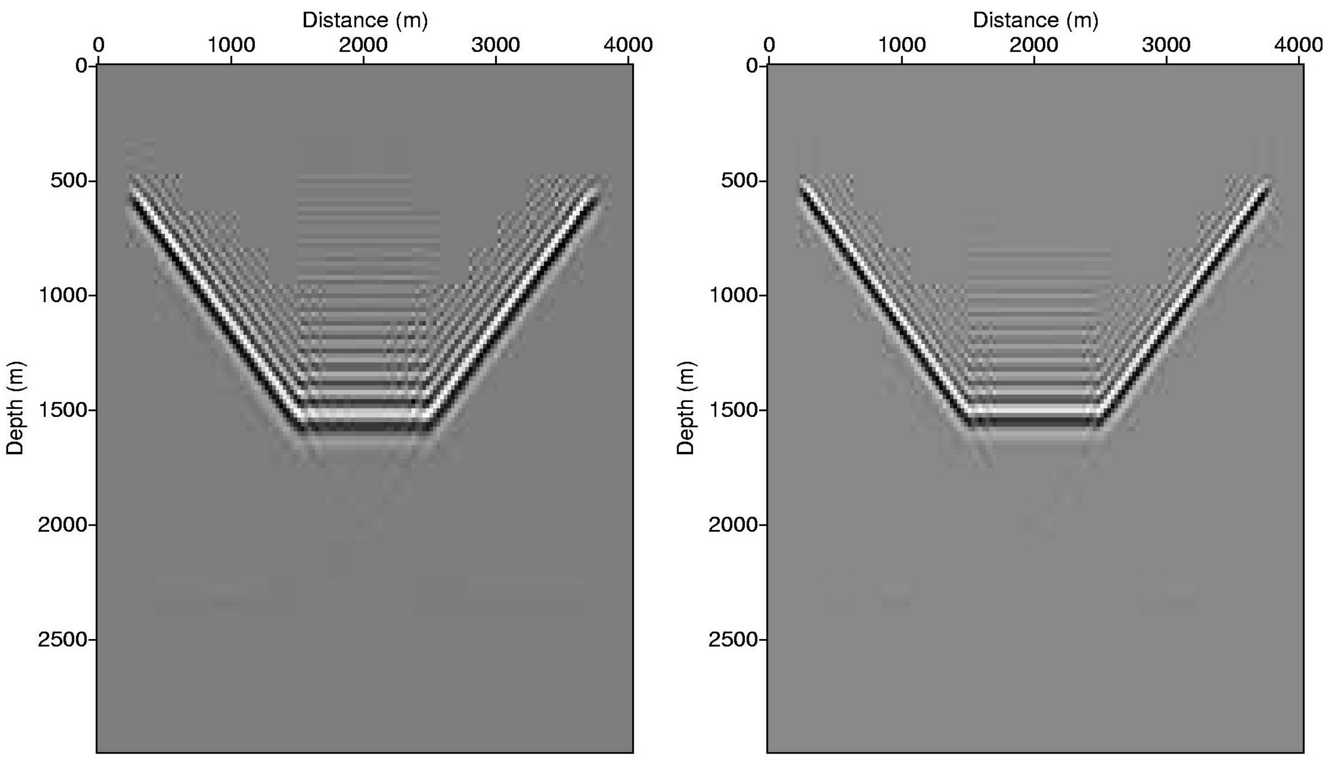


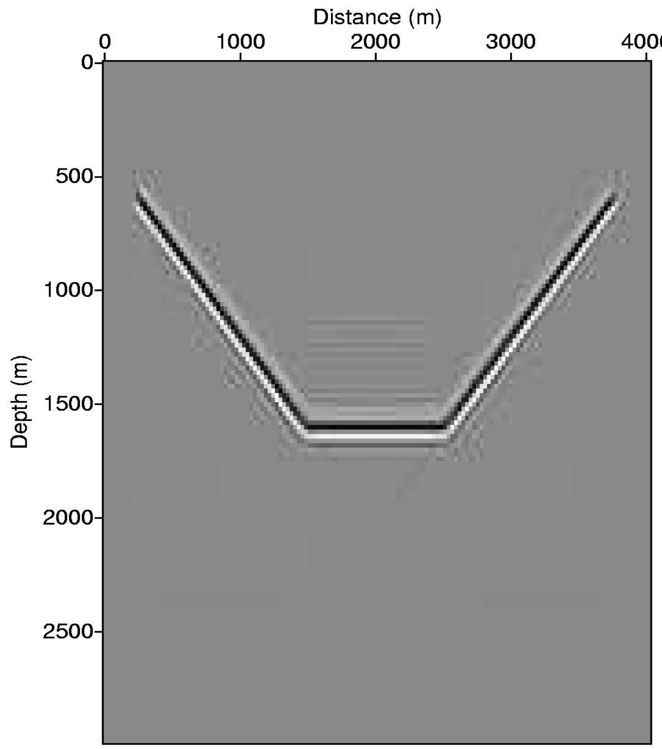
Figure 3 - Reverse-time migration results

(a) **Fourth order finite difference**

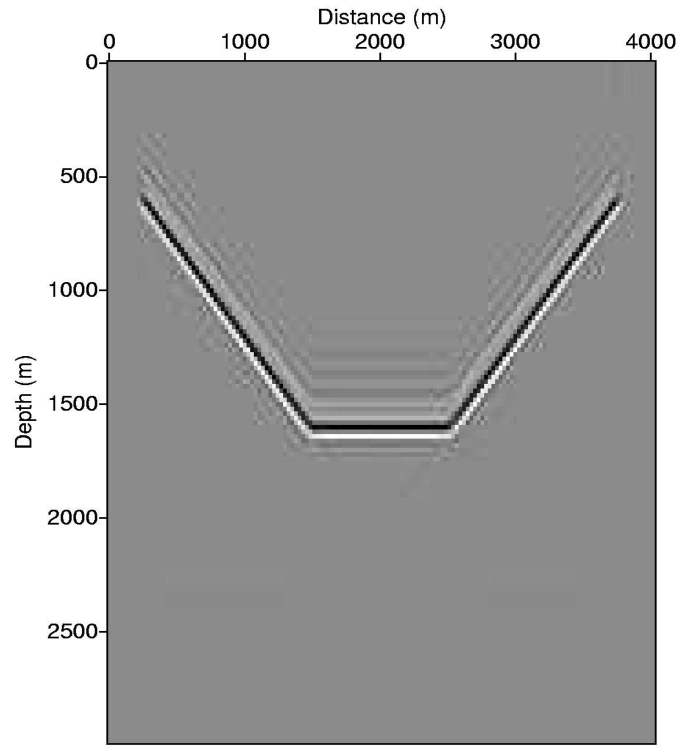
(b) **2-1 operator**



(c) **3-1 operator**



(d) **Pseudo spectral operator**



EDITED REFERENCES

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REFERENCES

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