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Summary

Migrated gathers are a key input for velocity model building and for subsurface characterization. Kinematic information is used for velocity model updates, and amplitude and phase attributes are used for the derivation of material properties. Conventional gathers are two dimensional: a vertical axis that is either time, time migrated or depth, and a horizontal axis that can be offset, or angle in the depth migrated domain. With wide azimuth acquisition the gathers are 3D by nature. The additional horizontal axis is the azimuth angle. The importance of the azimuthal information for short wavelength velocity determination has been demonstrated theoretically (Bartana et. al., 2009). The naïve way of describing the 3D gathers is by a set of azimuth sectors, where the offsets or angles are binned at each azimuthal sector. In other words, the separation for azimuth sectors treats the 3D gather as a set of multi 2D gathers. In this work we show a different approach, in which the azimuthal information is represented in a continuous manner for the purpose of delay analysis.

The method for representing the 3D angle gather and the delay analysis performed in this domain are described. The method is demonstrated on a synthetic example and on field data.

Introduction

With wide azimuth data the gathers are 3 dimensional by nature. For offset gathers, the offsets can be binned either by two offset coordinates: offsets in the x direction and offsets in the y direction; or by a set of discrete azimuth sectors, where the offsets are binned for each of the azimuth sectors. The two options for data organization can be used in the offset domain for the time gathers and for image gathers. The importance of the angle domain in depth imaging has been widely discussed (Koren et. al., 2011, Zhang et. al., 2011). In the angle domain the two horizontal coordinates in the 3D gather are the reflection angle and the azimuth of the reflection angle: γ_1 and γ_2 , respectively. In the angle domain the migrated gathers can be binned as multi-azimuth gathers, i.e. the traces are binned by reflection angle and a set of reflection angle azimuth sectors; or the traces can be arranged in reflection angle and azimuths in a continuous manner.

In this work the representation of the full azimuth 3D gathers is described. The delay analysis of the angle domain image gathers using the Poisson method is

described. The analysis method will be derived, and then demonstrated on synthetic data and field data.

3D angle gathers and delay analysis

When the data is represented as a set of azimuth sectors, the delay analysis is carried out in a 2D manner. The Plane Wave Destructor (PWD) is applied for each azimuth sector independently. For the full 3D analysis, the reflection angle and the azimuth angle are mapped into two local Cartesian coordinates x and y. The relation between the angles and the Cartesian coordinates:

1.
$$x = \gamma_1 \cos \gamma_2$$

2. $y = \gamma_1 \sin \gamma_2$

On the Cartesian grid, the depth delays are found as a surface in 3D volume $\tau(x, y)$. The surface $\tau(x, y)$ is found by solving the Poisson equation:

3.
$$\nabla^2 \tau = \frac{\partial^2 \tau}{\partial x^2} + \frac{\partial^2 \tau}{\partial y^2} = \frac{\partial p_x}{\partial x} + \frac{\partial p_y}{\partial y} = \underline{\nabla} \cdot \underline{p}$$

The vector $\underline{\mathbf{p}} = (p_{x_i}p_y)$ represents the data slopes obtained by applying the PWD method to the migrated gathers twice, in the x direction and in the y direction, respectively. The surface of the depth delays is represented on the Cartesian grid with two indices $\tau_{i,j}$, i in the range $[0..N_x - 1]$, and j in the range $[0..N_y - 1]$. The data slopes $\underline{\mathbf{p}}$ are given on a grid shifted by half sample $p_{x_{i+1/2,j}}, p_{y_{i,j+1/2}}$. The divergence of the slope vector is computed by finite difference:

4.
$$\left(\underline{\nabla} \cdot \underline{\mathbf{p}}\right)_{i,j} = \frac{1}{\Delta \mathbf{x}} \left(p_{x_{i+1/2,j}} - p_{x_{i-1/2,j}} \right) + \frac{1}{\Delta \mathbf{y}} \left(p_{y_{i,j+1/2}} - p_{y_{i,j-1/2}} \right)$$

Boundary conditions

Von Neumann boundary conditions are applied at the domain boundaries. A Dirichlet condition is imposed at the centre of the 3D gather, $\tau = 0$ at $\gamma_1 = 0$, i.e., the delay is defined as zero for the normal incident ray. For the Von Neumann boundary conditions, it is assumed that the second derivative vanishes at the boundaries. Consider the x direction of the Poisson equation at location (i = 0, j):

5.
$$\frac{1}{\Delta x^2} \left(\tau_{-1,j} + \tau_{1,j} - 2\tau_{0,j} \right) = 0$$

In other words, the slope in the x direction is constant at the edges of x:

6.
$$p_{x_{-1/2,j}} = p_{x_{0,j}} = p_{x_{1/2,j}}$$

The second order approximation for the first derivative at the boundary i=0:

7.
$$\frac{1}{2\Delta x} (\tau_{1,j} - \tau_{-1,j}) = p_{x_{0,j}}$$

Isolating the surface τ at location (-1,j) and inserting into equation 5:

8.
$$\frac{1}{\Delta x^2} (\tau_{1,j} - \tau_{0,j}) = \frac{1}{\Delta x} p_{x_{0,j}}$$

The same assumption is applied on the other side of the grid, $i = N_x - 1$, and at the boundaries in the y direction. This leads to the two dimensional finite difference Laplace operator. With the above boundary conditions the Laplace operator is represented by a sparse symmetric matrix, which enables solving the Poisson equation (3) by the conjugate gradient method. The resulting matrix has a block form. For a system where $\Delta x = \Delta y$ the block matrix reads:

9.
$$\begin{pmatrix} D_1 & I & . & . & . \\ I & D_2 & \ddots & . & . \\ . & I & \ddots & . & . \\ . & . & \ddots & I & . \\ . & . & . & D_2 & I \\ . & . & . & I & D_1 \end{pmatrix}$$

Dots represent zero blocks. The block matrix is $N_y \times N_y$. The dimension of the matrices D_1 , D_2 and I is $N_x \times N_x$. I is the unit matrix, D_1 , D_2 are given by:

10.
$$D_{1} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -3 & \ddots & 0 & 0 \\ 0 & 1 & \ddots & 1 & 0 \\ 0 & 0 & \ddots & -3 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix}$$
11.
$$D_{2} = \begin{pmatrix} -3 & 1 & 0 & 0 & 0 \\ 1 & -4 & \ddots & 0 & 0 \\ 0 & 1 & \ddots & 1 & 0 \\ 0 & 0 & \ddots & -4 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{pmatrix}$$

At the grid corners the diagonal element of the matrix equals (-2). Along the edges the diagonal equals (-3), at all other points the diagonal equals (-4). The central row and column of the matrix (9) are removed, as required by the Dirichlet condition.

The delay surface τ is obtained by an iterative solution of the Poisson equation $\nabla^2 \tau = \underline{\nabla} \cdot \underline{p}$. The Poisson equation is solved at each iteration using the conjugate gradient method. The iterative algorithm can be summarized:

- 1. In the first iteration the surface τ is assumed to be zero for all angles.
- 2. The PWD method is applied for obtaining the slope vector along the surface τ .
- 3. The right hand side is computed by finite difference, equation 4.
- 4. The Poisson equation is solved by the conjugate gradient method to obtain an updated surface τ .
- 5. Steps 2 to 4 are repeated until convergence.

Synthetic example

In the synthetic example the model involves 3 layers. The second layer is characterized by 4 small velocity anomalies in the correct model. The velocity model is shown in figure 1. A rose diagram of the wide azimuth acquisition pattern is displayed in figure 2. The data was migrated with the background velocity in the second layer. Since the imaging has been performed with an incorrect velocity, azimuthal dependent depth delays are expected close to the locations of the velocity anomalies. Figure 3 shows 12 sectors of angle gathers in a location close to one of the velocity anomalies. The azimuthal dependence depth delays are clearly seen. The picked delays were obtained by applying the PWD method to the set of 12 2D gathers. Figure 4 shows the 3D angle gather with the picked surface τ , at the same CDP location. The surface is colored by the value of the depth delays, showing the continuous variation of depth delays with azimuth.

Field data example

The field data is of a wide azimuth Barnett shale survey. Figure 5 shows the rose diagram of the acquisition pattern. Figure 6 shows the azimuthal dependent depth delays by 12 sectors of the angle gather. Figure 7 shows the 3D angle gather with the picked surface of depth delays at the same CDP location.

Conclusions

The 3D full azimuth description of the migrated gathers is used for delay analysis in the velocity model building process. The Poisson picker is used for the delay analysis. The continuous manner of dealing with the azimuthal information is very important when dealing with complex velocity models and when anisotropy is involved, especially azimuthal anisotropy.

a)

Acknowledgements

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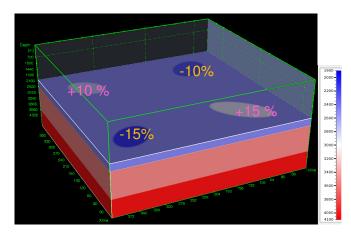
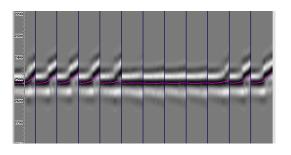


Figure 1. Velocity model of the synthetic data. In the correct model 4 velocity anomalies are present in the second layer. Migration has been performed with the background velocity of the second layer.



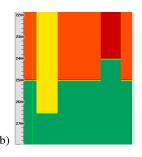


Figure 3. a) Twelve azimuthal sectors in the angle domain, with a 2D auto-picker (purple line). b) A section of the correct velocity field. The CMP location of the image gather is marked by the vertical green dashed line.

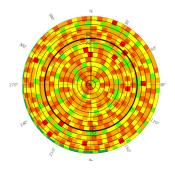


Figure 2. Rose diagram of the acquisition geometry for the synthetic data.

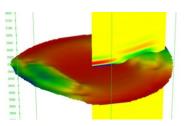


Figure 4. 3D angle gather in the continuous azimuth representation. The "section" represents an azimuth sector. The surface represents the surface τ obtained by solving the Poisson equation for this data. The surface is color coded by the delay value. Green color represents negative delays.

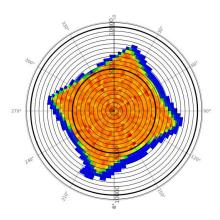


Figure 5. Rose diagram of the acquisition geometry of the Barnett shale survey.

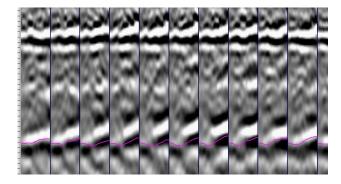


Figure 6. Twelve azimuthal sectors in the angle domain. The 2D auto-picker is displayed by the purple line.

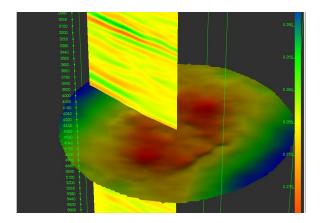


Figure 7. 3D angle gather in the continuous azimuth representation. The "section" represents the gather in azimuth 60° . The surface represents the surface τ obtained by solving the Poisson equation for this data. The surface is color coded by the delay value. Blue color represents negative delays.

EDITED REFERENCES

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