

Short Note

A nonreflecting boundary condition for discrete acoustic and elastic wave equations

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INTRODUCTION

One of the nagging problems which arises in application of discrete solution methods for wave-propagation calculations is the presence of reflections or wraparound from the boundaries of the numerical mesh. These undesired events eventually override the actual seismic signals which propagate in the modeled region.

The solution to avoiding boundary effects used to be to enlarge the numerical mesh, thus delaying the side reflections and wraparound longer than the range of times involved in the modeling. Obviously this solution considerably increases the expense of computation. More recently, nonreflecting boundary conditions were introduced for the finite-difference method (Clayton and Enquist, 1977; Reynolds, 1978). These boundary conditions are based on replacing the wave equation in the boundary region by one-way wave equations which do not permit energy to propagate from the boundaries into the numerical mesh. This approach has been relatively successful, except that its effectiveness degrades for events which impinge on the boundaries at shallow angles. It is also not clear how to apply this type of boundary condition to global discrete methods such as the Fourier method for which all grid points are coupled.

In this note we describe an alternative scheme for constructing a nonreflecting boundary condition. It is based on gradual reduction of the amplitudes in a strip of nodes along the boundaries of the mesh. The method appears extremely simple and robust, and it can be applied to a wide variety of time-dependent problems. Unlike other methods, the effectiveness of this boundary condition does not decrease for shallow angles of incidence.

In the following sections we briefly describe the nonreflecting boundary condition and demonstrate its application to the Fourier method for acoustic and elastic wave propagation.

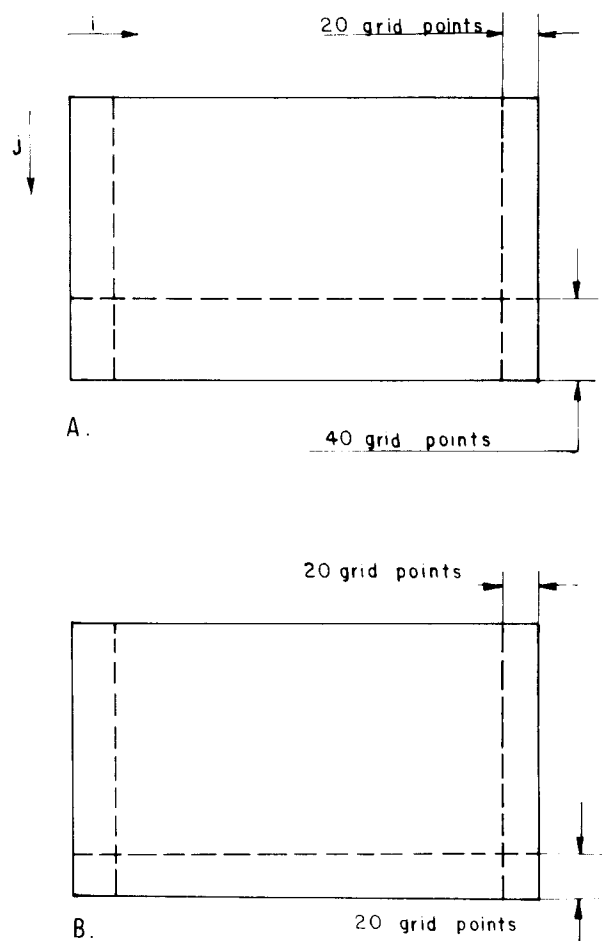


FIG. 1. (a) Grid configuration for nonreflecting boundary condition without free surface. (b) Grid configuration for problems with free surface.

Manuscript received by the Editor November 9, 1983; revised manuscript received October 9, 1984.

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METHOD

We illustrate the nonreflecting boundary condition for the Fourier method of solving the acoustic wave equation (Kosloff and Baysal, 1982). Let $P^n(i, j)$ denote the pressure at time $t = nDt$ and at spatial location $x = iDx$, $y = jDy$, $i = 1 \dots Nx$, $j = 1 \dots Ny$, where dt denotes the time step size and Dx and Dy denote the mesh size in the x - and y -directions. A typical time step of the solution method runs as follows (Kosloff and Baysal, 1982):

- (1) calculate $R^n(i, j) = \partial^2 P^n / \partial x^2 + \partial^2 P^n / \partial y^2$ by the Fourier approximation,
- (2) integrate in time according to

$$\dot{P}^{n+1/2}(i, j) = \dot{P}^{n-1/2}(i, j) + Dt R^n(i, j) C^2(i, j),$$

and

$$P^{n+1}(i, j) = P^n(i, j) + Dt \dot{P}^{n+1/2}(i, j),$$

where $C(i, j)$ denotes the acoustic velocity.

The calculations in steps (1) and (2) are repeated for the desired number of time steps.

For the nonreflecting boundary condition the values of P^{n+1} and $\dot{P}^{n+1/2}$ are slightly reduced after each time step in a strip of nodes surrounding the numerical mesh (Figures 1a, 1b). For the Fourier method we found a strip width of twenty nodes sufficient to reduce side reflections to a few percent. The amplitude

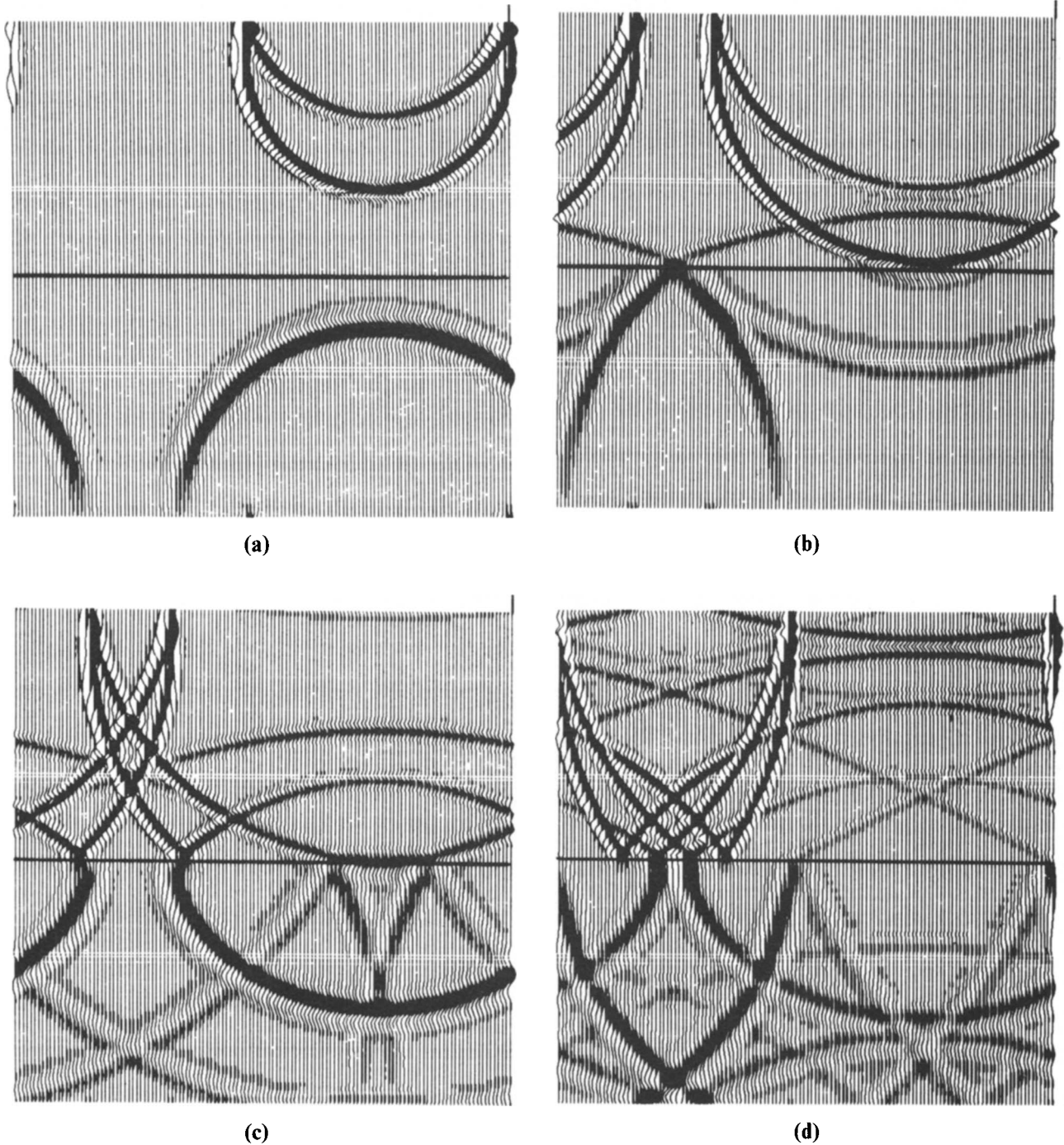


FIG. 2. Amplitudes at progressive times for acoustic wave propagation without nonreflecting boundaries.

reduction in each strip is gradually tapered from a zero value in the interior boundary. For example, for the strip on the left portion of Figure 1 ($1 \leq i \leq 20$) the pressure amplitudes are multiplied by the factor $G = \exp - [0.015(20 - i)]^2$. This gives a value of 1 for $i = 20$ and a value of 0.92 for $i = 1$. For the lower boundary there is a distinction between problems without a free-surface boundary condition (Figure 1a) and problems which include a free-surface boundary condition (Figure 1b). To avoid wraparound in the former, the lower strip has a width of 40 with maximum amplitude reduction in the center of the strip. For free-surface problems the lower strip width is 20 (Figure 1b) with maximum reduction on the boundary.

For elastic wave calculations the nonreflecting boundary conditions can be affected by reducing the magnitudes¹ of the field variables at the end of each time step. For the stress equation method (Kosloff et al., 1983) the variables include the stress components and the stress-time derivatives.

¹Phase changes are not mentioned here but are not considered relevant by the authors "since phase changes cannot be noticed if the amplitudes are eliminated successfully by the boundary condition."—Ed.

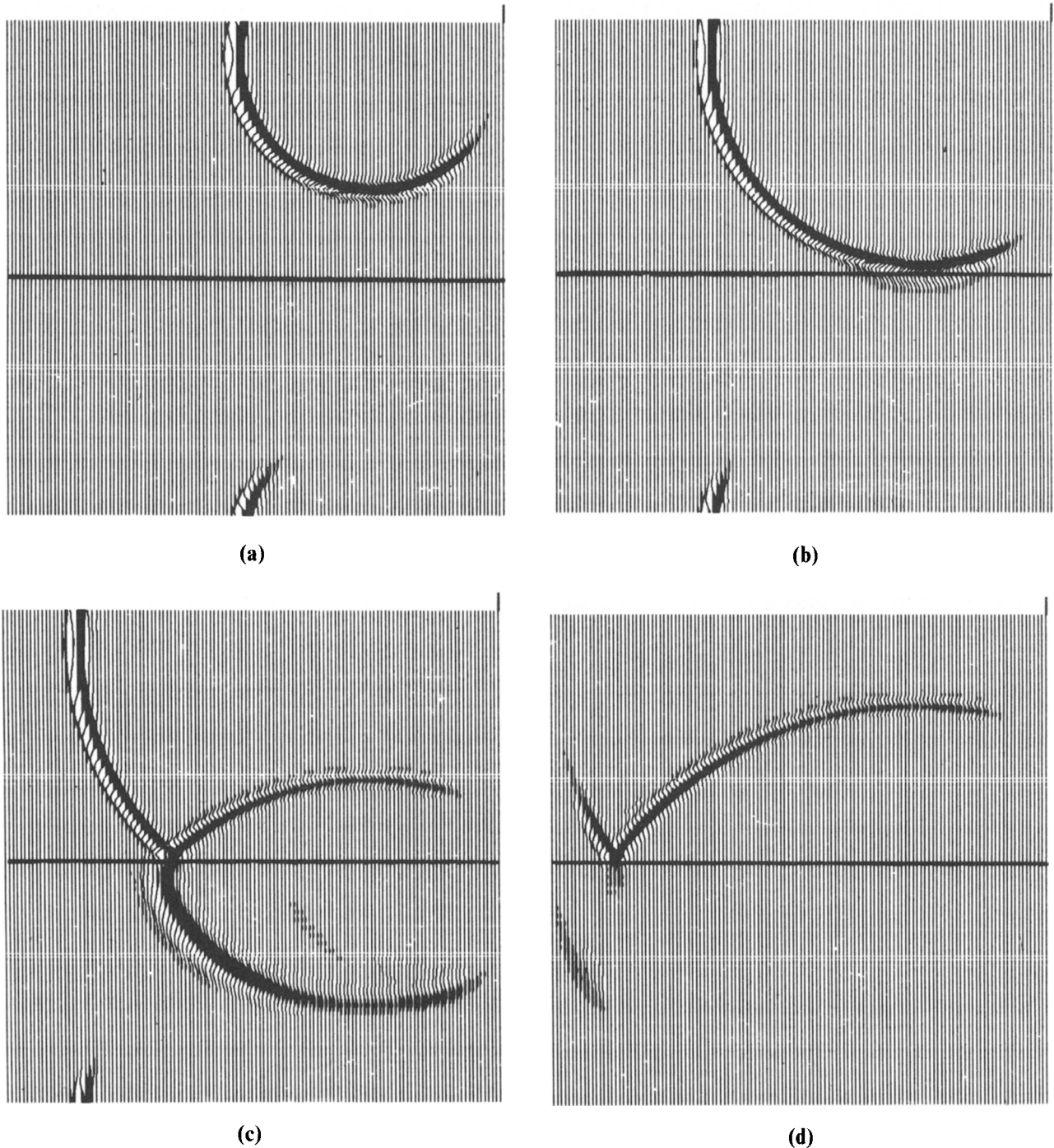


FIG. 3. Amplitudes at progressive times for acoustic wave propagation with nonreflecting boundaries.

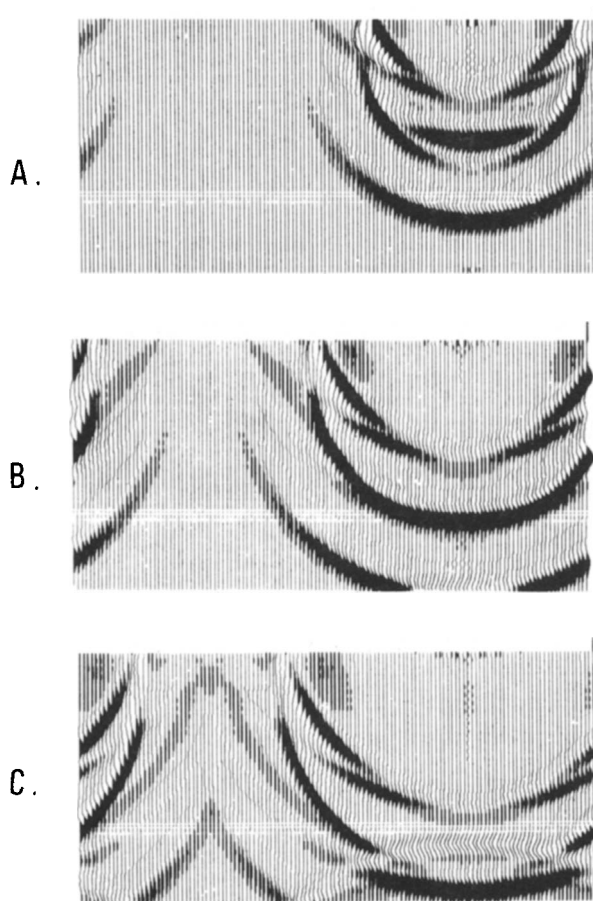


FIG. 4. Amplitudes for elastic wave propagation without nonreflecting boundaries.

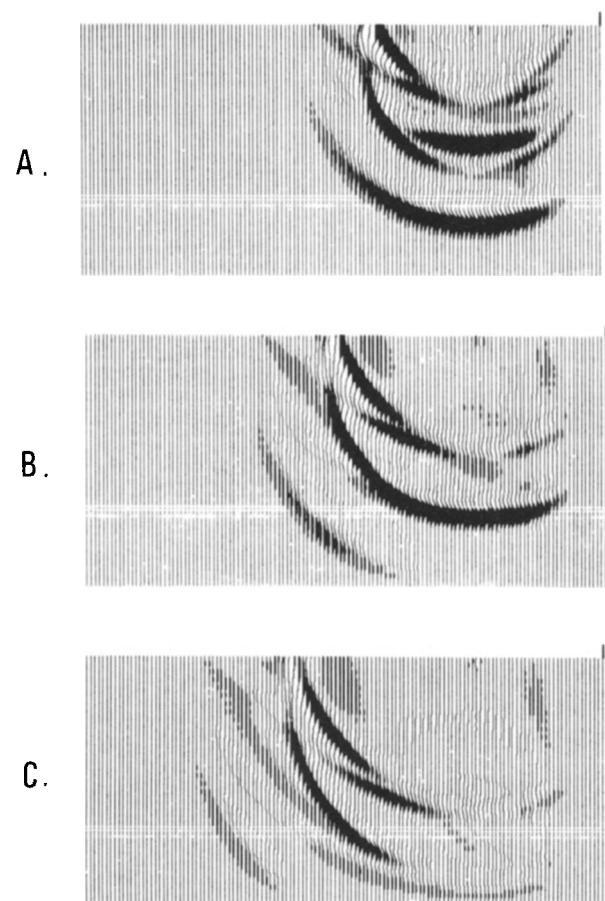


FIG. 5. Amplitudes for elastic wave propagation with nonreflecting boundaries.

EXAMPLES

The first example includes acoustic wave propagation in a medium without a free surface containing two velocity regions in planar contact. The velocities were 2.5 km/s for the upper region and 4.0 km/s for the lower region. A band-limited point source was applied in the vicinity of the free surface. The calculation used a 128×128 grid with a grid size $Dx = 20$ m and a time step size of 1 ms.

Figures 2a–2d present amplitude snapshots at progressive times. The late snapshots are cluttered by wraparound events. Figures 3a–3d show the corresponding snapshots which were obtained with the nonreflecting boundary condition. In the central portion of the grid these snapshots show the true events only and the wraparound events are eliminated before they can reenter the central part of the grid.

The second example is of elastic wave propagation in a uniform region with a free surface. The P and S velocities were, respectively, 4.0, 2.4 km/s and the calculations used a 128×64 grid with a grid size of 20 m. A point vertical force was applied slightly below the surface. Figures 4a–4c and Figures 5a–5c, respectively, compare snapshots which were obtained with and without the nonreflecting boundary conditions. The two sets of snapshots for this example give the same conclusions as in the previous acoustic example.

CONCLUSIONS

We have presented a simple method to effect a nonreflecting boundary condition for direct methods of solving wave propagation problems. The boundary condition was demonstrated for the Fourier method for solving acoustic and elastic wave propagation. The method can also be used after slight modification with explicit finite-element or finite-difference techniques. Our experience with the method so far indicates that it can be applied to a wide variety of acoustic and elastic problems, even including elastic calculations for models which contain fluids and solids in juxtaposition (Fisher, 1983).

REFERENCES

- Clayton, R., and Enquist, B., 1977, Absorbing boundary conditions for acoustic and elastic wave equations: *Bull., Seis. Soc. Am.*, **67**, 1529–1540.
- Fisher, D. A., 1983, Complex seismic trace attributes and source receiver attribute: *Seismic Acoustics Laboratory 6th Year Annual Progress Review*, University of Houston.
- Kosloff, D., and Baysal, E., 1982, Forward modeling by a Fourier method: *Geophysics*, **47**, 1402–1412.
- Kosloff, D., Reshet, M., and Loewenthal, D., 1983, Elastic forward modeling: *Bull., Seis. Soc. Am.*, **74**, 875–891.
- Reynolds, A. C., 1978, Boundary conditions for the numerical solution of wave propagation problems: *Geophysics*, **43**, 1099–1110.